Important length scales in dense gas-particle flows

Stefan Radl, Chris Milioli, Fernando Milioli & Sankaran Sundaresan
Princeton University

Paper 172c, 03B09 Special session to celebrate
John Chen’s career long accomplishments
Monday, October 29, 2012
Conference C (Omni)
Outline

• Background on dimensional analysis of fluidized beds
• Characteristic length at small scales
• Characteristic length at larger scales
• Proposed hierarchy of importance for the dimensionless groups in scale-up and scale-down of turbulent fluidized beds and CFB risers
Fluidized bed scale-up or scale-down

- What are important dimensionless groups that must be matched while scaling turbulent and fast fluidized beds?
  - **Dimensional quantities:**
    - Densities of gas (at inlet) and solid \( (\rho_g \text{ and } \rho_s) \)
    - Mass fluxes of gas (at inlet) and solid \( (G_g \text{ and } G_s) \)
    - Particle diameter, \( d_p \) (more accurately, PSD)
    - Gas viscosity, \( \mu_g \)
    - System size, \( D \)
    - Gravitational constant, \( g \), matters, but is fixed.
  - **Extent of gas density variation as it flows through the bed**
    - Low pressure vs. high pressure
  - **Microscopic, but dimensionless, parameters:**
    - Friction coefficient and coefficient of restitution
      - Particle-particle: \( \mu_p \text{ and } e_p \)
      - Particle-wall: \( \mu_w \text{ and } e_w \)
    - Particle shape, asperity

\[
G_i = \rho_i U_i, \quad i = s, g
\]
Dimensional analysis

- **Dimensionless groups**
  - Gas and solid velocity ratio
  - Gas and solid density ratio
  - Particle-to-bed size ratio
  - Froude number (bed-scale)
  - Froude number (particle scale)
  - Particle Reynolds number

\[
\frac{U_g}{U_s}, \frac{\rho_g}{\rho_s}, \frac{d_p}{D}, Fr_b = \frac{U_g^2}{gD}, \quad Fr_p = \frac{v_t^2}{g d_p}, \quad Re_p = \frac{\rho_g v_t d_p}{\mu_g}, \quad PSD, Geometry
\]

\[
Ar = \left(\frac{\rho_s}{\rho_g} - 1\right) \frac{\rho_g^2 g d_p^3}{\mu_g^2} = \left(\frac{\rho_s}{\rho_g} - 1\right) \frac{Re_p^2}{Fr_p}, \quad \bar{Re} = \frac{\rho_g g d_p^{3/2}}{\mu_g} = \frac{Re_p}{Fr_p^{1/2}}
\]

\[
Fr = \frac{U_g^2}{gd_p} = Fr_b \left(\frac{d_p}{D}\right)^{-1}
\]

\[
\frac{U_g}{U_s}, \frac{\rho_g}{\rho_s}, \frac{d_p}{D}, \quad Fr, Ar, \bar{Re}, PSD, Geometry
\]

- **If only viscous drag**

  \[
  \mu_w, e_w, \mu_p, e_p
  \]

  Cannot match all of these
Use model equations to examine scaling

- **Glicksman (1984, 1994,...)**
  - Two-fluid model
    - continuity and momentum balance
    - Neglect particle phase stress

\[
\frac{U_g}{U_s}, \frac{\rho_g}{\rho_s}, \frac{d_p}{D}, \text{Fr, Ar, } \text{Re, PSD, Geometry}
\]

- **Detamore et al. (2001)**
  - Kinetic theory model, steady state simulations
  - Must pay attention to particle interaction parameters: \( e_w, e_p \)
Use model equations to examine scaling

  - Hydrodynamic similarity of riser flows
  - Global quantities match if we match:
    - Nothing related to particle diameter or gas viscosity!

- Naren & Ranade (Particuology, 9, 121-129, 2011)
  - Tested through simulations
  - Yes, but radial profiles don’t match
Objective of our study

- Dimensionless groups:
  \[ \frac{U_g}{U_s}, \frac{\rho_g}{\rho_s}, \frac{d_p}{D}, \quad Fr_b = \frac{U_g^2}{gD}, \quad Fr_p = \frac{v_i^2}{gd_p}, \quad Re_p = \frac{\rho_g v_i d_p}{\mu_g}, \quad \text{PSD, Geometry} \]

- Group these in terms of their sphere of influence
  - Important for similarity of macro-scale flow characteristics
  - Influence limited to smaller scales

- Goal: Understand what terms (in the two-fluid model) are competing at different scales?
Two-fluid model: Momentum balance

\[
\frac{\partial}{\partial t} \left( \rho_s \phi_s \mathbf{v}_s \right) + \nabla \cdot \left( \rho_s \phi_s \mathbf{v}_s \mathbf{v}_s \right) = -\nabla \cdot \mathbf{\sigma}_s - \phi_s \nabla \cdot \mathbf{\sigma}_g + \beta (\mathbf{v}_g - \mathbf{v}_s) + \rho_s \phi_s \mathbf{g}
\]

- Principally viscous drag – good approx. for most TFB and CFB

\[
\beta = \frac{\mu_g}{d_p^2 v_t} f_1(\phi_s) = \frac{\rho_s g}{v_t} f_2(\phi_s)
\]

\[
\frac{\partial}{\partial t} \left( \rho_g \phi_g \mathbf{v}_g \right) + \nabla \cdot \left( \rho_g \phi_g \mathbf{v}_g \mathbf{v}_g \right) = -\phi_g \nabla \cdot \mathbf{\sigma}_g - \beta (\mathbf{v}_g - \mathbf{v}_s) + \rho_g \phi_g \mathbf{g}
\]

- Let us cast these in dimensionless form
Two-fluid model: Momentum balance

\[
\frac{\partial}{\partial t} \left( \rho_s \phi_s \mathbf{v}_s \right) + \nabla \cdot \left( \rho_s \phi_s \mathbf{v}_s \mathbf{v}_s \right) = -\nabla \cdot \sigma_s - \phi_s \nabla p_g + \phi_s \nabla \cdot \tau_g + \frac{\rho_s g}{V_t} f_2(\phi_s) (\mathbf{v}_g - \mathbf{v}_s) + \rho_s \phi_s g
\]

- In fluidized flows, drag nearly balances the weight of the particles. So, \( v_t \) is a natural characteristic velocity
- Particle density is a natural density scale
- What about length scale? Many choices!!
  - Inertia \( \sim \) Gravity

\[
L_1 \sim \frac{v_t^2}{g}
\]
Two-fluid model: Particle phase stress

\[ \frac{\partial}{\partial t} (\rho_s \phi_s \mathbf{v}_s) + \nabla \cdot (\rho_s \phi_s \mathbf{v}_s \mathbf{v}_s) = - \nabla \cdot \mathbf{\sigma}_s - \phi_s \nabla p_g + \phi_s \nabla \cdot \mathbf{\tau}_g + \frac{\rho_s g}{v_t} f_2(\phi_s)(\mathbf{v}_g - \mathbf{v}_s) + \rho_s \phi_s \mathbf{g} \]

- Particle phase stress:
  \[ \mu_s \sim \rho_s v_t d_p \]

- Particle phase deviatoric stress \( \sim \) Gravity
  \[ \mu_s \sim \rho_s T^{1/2} d_p \]

- Particle phase deviatoric stress \( \sim \) Inertia
  \[ L \sim \frac{v_t^2}{g} (Fr_p)^{-n}, \quad n = \frac{1}{2}, \frac{2}{3}, -1, -\frac{4}{3} ; \quad Fr_p = \frac{v_t^2}{gd_p} \]
In fluidized flows, drag nearly balances the weight of the particles. So, $v_t$ is a natural characteristic velocity.

Particle density is a natural density scale.

What about length scale?

Each corresponds to competition between different pairs.

Question: As one examines progressively larger scale, how does this competition shift?
**System 1**

**Given:** \( \rho_{s1}, d_{p1}, \rho_{g1}, \mu_{g1} \)

**Periodic box size:** \( \Delta_{d1} \times \Delta_{d1} \times 4\Delta_{d1} \)

**Fluid grid size:** \( \Delta_{f1} \)

Determine dimensionless domain-average slip velocity:

\[
\frac{\langle V_{\text{slip}, 1} \rangle}{V_{t1}}
\]

\[
\frac{\rho_{s1}}{\rho_{g1}} = 1500 / 1.3 \text{ [kg/m}^3]\]

\[
\mu_{g1} = 1.8 \cdot 10^{-5} \text{ [Pa} \cdot \text{s]}\]

\[
\Delta_{d1} / \Delta_{f1} = 4 / 0.25 \text{ [mm]}\]

\[
d_{p1} = 75 \mu\text{m}\]

\[
\phi_s = 0.05 (0.25)\]

\[
v_{t1}^2/g = 4.86 \text{ [mm]}\]

\[
Fr_{p1} = 64.9\]

(\( \phi_s = 0.25 \), particles colored according local particle volume fraction)
How should we scale the CFD-DEM simulations?

**System 1**
Given: \( \rho_{s1}, d_{p1}, \rho_{g1}, \mu_{g1} \)
Periodic box size: \( \Delta_{d1} \times \Delta_{d1} \times 4\Delta_{d1} \)
Fluid grid size: \( \Delta_{f1} \)
Determined dimensionless domain-average slip velocity

**System 2**
Given: \( \rho_{s2}, d_{p2}, \rho_{g2}, \mu_{g2} \)
Periodic box size: \( \Delta_{d2} \times \Delta_{d2} \times 4\Delta_{d2} \)
Fluid grid size: \( \Delta_{f2} \)

Want:

\[
\frac{\langle V_{\text{slip}, 2} \rangle}{V_{t2}} = \frac{\langle V_{\text{slip}, 1} \rangle}{V_{t1}}
\]

\[
\frac{\Delta_{d2}}{L_{d2}} = \frac{\Delta_{d1}}{L_{d1}}
\]
\[
\frac{\Delta_{f2}}{L_{f2}} = \frac{\Delta_{f1}}{L_{f1}}
\]

\[
L_d \sim \frac{V_t^2}{g \left( Fr_p \right)^{-n_d}} \quad n_d = 0, -\frac{1}{2}, -\frac{2}{3}, -1, -\frac{4}{3}
\]
\[
L_f \sim \frac{V_t^2}{g \left( Fr_p \right)^{-n_f}} \quad n_f = 0, -\frac{1}{2}, -\frac{2}{3}, -1, -\frac{4}{3}
\]

Which of these make sense?
How should we scale the CFD-DEM simulations?

**System 1**

**Given:** \( \rho_{s1}, d_{p1}, \rho_{g1}, \mu_{g1}, g_1 \)

**Periodic box size:** \( \Delta_d x \Delta_d x 4\Delta_d \)

**Fluid grid size:** \( \Delta_{f1} \)

**System 2**

**Given:** \( \rho_{s2}, d_{p2}, \rho_{g2}, \mu_{g2}, g_2 \)

**Periodic box size:** \( \Delta_d x \Delta_d x 4\Delta_d \)

**Fluid grid size:** \( \Delta_{f2} \)

**Want:**

\[
\frac{\Delta_{d2}}{L_{d2}} = \frac{\Delta_{d1}}{L_{d1}}
\]

\[
\frac{\Delta_{f2}}{L_{f2}} = \frac{\Delta_{f1}}{L_{f1}}
\]

\[
\frac{\left\langle V_{slip,2} \right\rangle}{V_{t2}} = \frac{\left\langle V_{slip,1} \right\rangle}{V_{t1}}
\]

\[
L_d \sim \frac{V_t^2}{g \left( Fr_p \right)^{-n_d}}, \quad n_d = 0, -\frac{1}{2}, -1, -\frac{4}{3}
\]

\[
L_f \sim \frac{V_t^2}{g \left( Fr_p \right)^{-n_f}}, \quad n_f = 0, -\frac{1}{2}, -1, -\frac{4}{3}
\]
How should we scale the CFD-DEM simulations?

- At “small scale”, both resolution requirement and filtered statistics scale as

\[
\frac{v^2}{g} \left( \frac{Fr_p}{R} \right)^{-2/3}
\]

What physical picture does this imply?

- Particle phase stress - kinetic theory:

\[
\mu_s \sim \rho_s T^{1/2} d_p
\]

- Plus, Particle phase deviatoric stress \(\sim\) Gravity

\[
\left[ \frac{\partial}{\partial t} \left( \rho_s \phi_s v_s \right) + \nabla \cdot \left( \rho_s \phi_s v_s v_s \right) \right] = -\nabla \cdot \sigma_s - \phi_s \nabla p_g + \phi_s \nabla \cdot \tau_g + \frac{\rho_s g}{v_t} f_2(\phi_s)(v_g - v_s) + \rho_s \phi_s g
\]
Grid resolution requirement of TFM simulations

- At “small scale”, both resolution requirement and filtered statistics scale as

\[ \frac{v_t^2}{g} \left( \frac{F_r}{p} \right)^{-2/3} \]

- Particle phase stress - kinetic theory:

\[ \mu_s \sim \rho_s T^{1/2} d_p \]

- Grid resolution requirement of TFM is set by the need to resolve the competition between drag and viscous forces!

\[
\begin{bmatrix}
\frac{\partial}{\partial t} (\rho_s \phi_s v_s) + \nabla \cdot (\rho_s \phi_s v_s v_s)
\end{bmatrix} = -\nabla \cdot \sigma_s - \phi_s \nabla p_g + \phi_s \nabla \cdot \tau_g + \frac{\rho_s g}{v_t} f_2(\phi_s)(v_g - v_s) + \rho_s \phi_s g
\]
Coarser structures resolved in KT-TFM simulations

• What is the correct length scale for the filter size?

\[ L_{\text{filter}} \sim \frac{v_t^2}{g} \left( Fr_p \right)^{-n_{\text{filter}}} \], \( n_{\text{filter}} = ? \)

• Grid resolution requirement of TFM is set by the need to resolve the competition between drag and viscous forces!

\[ L_{\text{grid}} \sim \frac{v_t^2}{g} \left( Fr_p \right)^{-2/3} \]

\[ \Delta_{\text{grid}} << \Delta_{\text{fil}} << \Delta_{\text{domain}} \]
Coarser structures resolved in KT-TFM simulations

- **What is the correct length scale for the filter size?**

\[ L_{\text{filter}} \sim \frac{V_t^2}{g} \left( Fr_p \right)^{-n_{\text{filter}}}, n_{\text{filter}} = ? \]

- **How did we test?:** Perform simulations with different particle sizes (75 – 300 μm), scaling the filter and domain sizes using different \( n_{\text{filter}} \) values.

\[ \frac{\Delta_{\text{grid}}}{L_{\text{grid}}} = \text{fixed} \]

\[ L_{\text{grid}} \sim \frac{V_t^2}{g} \left( Fr_p \right)^{-2/3} \]

\[ \Delta_{\text{grid}} \ll \Delta_{\text{fil}} \ll \Delta_{\text{domain}} \]

Domain-averaged particle volume fraction = 0.15
Coarser structures resolved in KT-TFM simulations

- What is the correct length scale for the filter size?

\[ L_{\text{filter}} \sim \frac{V_t^2}{g} \left( Fr_p \right)^{-n_{\text{filter}}} \], \( n_{\text{filter}} = ? \)

- Examine resulting filtered particle phase stress and filtered fluid-particle drag coefficient

\[ \frac{\Delta_{\text{grid}}}{L_{\text{grid}}} = \text{fixed} \]

\[ L_{\text{grid}} \sim \frac{V_t^2}{g} \left( Fr_p \right)^{-2/3} \]

\[ \Delta_{\text{grid}} << \Delta_{\text{fil}} << \Delta_{\text{domain}} \]

Domain-averaged particle volume fraction = 0.15
Dimensionless filtered particle phase viscosity

\[ L_{\text{filter}} \sim \frac{V^2}{g} \left( Fr_p \right)^{-n_{\text{filter}}} \]

\[ \Delta_{\text{filter}} / L_{\text{filter}} = \text{constant, but larger} \]

\[ \mu_{s, \text{fil}} / \left( \rho_s v_t L_{\text{filter}} \right) \]

Compare results obtained with different dimensional particle sizes, and subsequently cast in dimensionless form

\[ \left[ \frac{\partial}{\partial t} \left( \rho_s \phi_s v_s \right) + \nabla \cdot \left( \rho_s \phi_s v_s v_s \right) \right] = -\nabla \cdot \sigma_s - \phi_s \nabla p_g + \phi_s \nabla \cdot \tau_g + \frac{\rho_s g}{v_t} f_2(\phi_s)(v_g - v_s) + \rho_s \phi_s g \]
What does it all mean?

\[
\frac{\partial}{\partial t} (\rho_s \phi_s \mathbf{v}_s) + \nabla \cdot (\rho_s \phi_s \mathbf{v}_s \mathbf{v}_s) = -\nabla \cdot \sigma_s - \phi_s \nabla \cdot \sigma_g + \frac{\rho_s g}{v_t} f_2(\phi_s)(\mathbf{v}_g - \mathbf{v}_s) + \rho_s \phi_s g
\]

\[
\frac{\partial}{\partial t} (\rho_g \phi_g \mathbf{v}_g) + \nabla \cdot (\rho_g \phi_g \mathbf{v}_g \mathbf{v}_g) = -\phi_g \nabla \cdot \sigma_g - \frac{\rho_s g}{v_t} f_2(\phi_s)(\mathbf{v}_g - \mathbf{v}_s) + \rho_g \phi_g g
\]

- For TFBs and CFB risers, the particle phase stress and the deviatoric part of the fluid phase stress have little influence on coarse structures. \( \nabla \cdot \sigma_g \sim \nabla p_g \)

- Now consider a real physical system

- Characteristic length for macro-scale structures in TFB/CFB riser is \( \frac{U_g^2}{g} \).
What does it all mean?

\[ U_g^2 / gD \]

is an important dimensionless group to match the macro-scale characteristics

\[ U_s / U_g \]

immediately arises via flux specification

  - Hydrodynamic similarity of riser flows
  - Global quantities match if we match:
    - \( \left( \frac{U_g^2}{gD} \right)^{-0.3} \frac{U_s}{U_g} \)
  - Nothing related to particle diameter or gas viscosity!
What does it all mean?

$U_g^2 / gD$ is an important dimensionless group to match the macro-scale characteristics.

$U_s / U_g$ immediately arises via flux specification.

$U_g / v_t$ arises via correction to the drag force resulting from small scale structures (clusters and streamers).

$\rho_g / \rho_s$ may enter only via global arguments.

Usually $\rho_g \phi_g << \rho_s \phi_s$.

Let $\rho_g = Kp_g$. Then, if $\frac{\rho_s}{\rho_g} KgH << 1$, the density ratio is not important.

(Violated by deep beds at low pressures!)
Revisited scale-up and scale-down.

Using a combination of CFD-DEM and TFM simulations, we have examined the “competition” between terms in the momentum balance equations.

Model analysis suggest the following hierarchy:

Most important: \( \frac{U_g^2}{gD}, \frac{U_s}{U_g} \)

Next important: \( \frac{U_s}{v_t} \)

Then: \( \frac{\rho_s K}{\rho_g H}, \) when it is not much smaller than 1.

For systems with \( \rho_g \phi_g \ll \rho_s \phi_s \).
Acknowledgments

Funding from:

• S. Radl: Erwin-Schrödinger fellowship
• Milioli: FAPESP - São Paulo State Research Foundation (Brazil)
• ExxonMobil Research & Engineering Co.