Constitutive model for the fluid-particle drag coefficient in filtered two-fluid models for gas-particle flows

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Filtered two-fluid model: Overview

MICRO-SCALE ~ 50μm-mm

MESO-SCALE ~ mm-cm

MACRO-SCALE ~ cm-m

DEM for particles, DNS or CFD of averaged equations for the fluid flow.

Volume-averaged hydrodynamic models for fluid and particle phases.

Filtered volume-averaged hydrodynamic models for fluid and particle phases.

Engineering need: tools to probe macro-scale flow features directly.

Courtesy: Franklin Shaffer, NETL, Morgantown, WV (2009)

High Speed Particle Imaging: Riser Flow
Filtered two-fluid model: Overview

Develop models that allow us to focus on large-scale flow structures, without ignoring the possible consequence of the smaller scale structures.

Original two-fluid model and constitutive relations

* Significant advances in the past three decades

Modified constitutive relations for hydrodynamic terms
species and energy dispersion*
interphase heat and mass transfer rates*
even modified reaction rate expressions!

Filtered two-fluid model: Overview

MICRO-SCALE ~ 50μm-mm

MESO-SCALE ~ mm-cm

MACRO-SCALE ~ cm-m

Approach: Probe details of meso-scale structures and develop effective coarse-grained equations
Filtered two-fluid model: Overview

Filter “data” generated through highly resolved simulations of two-fluid models

- Snapshot of particle volume fraction field – kinetic theory based two-fluid model.
- Squares of different sizes illustrate regions (i.e. filters) of different sizes.

\[
\tilde{V}_g = \frac{\phi_g V_g}{\phi_g} ; \quad \tilde{V}_s = \frac{\phi_s V_s}{\phi_s}
\]

\(\Delta_{\text{grid}} \ll \Delta_{\text{fil}} \ll \Delta_{\text{domain}}\)

Igci et al., (2008)
Filtered drag coefficient

Filter “data” generated through highly resolved simulations of two-fluid models

- Snapshot of particle volume fraction field – kinetic theory based two-fluid model.
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\[
\beta_{fil} = \frac{\beta_{micro} \left( \phi_s, |v_g - v_s| \right) (v_{gy} - v_{sy}) - \phi'_s \frac{\partial p'_g}{\partial y}}{\left( \tilde{v}_{gy} - \tilde{v}_{sy} \right)}
\]

\[\Delta_{grid} << \Delta_{fil} << \Delta_{domain}\]

Igci et al., (2008)
Filtered drag coefficient

Filter “data” generated through highly resolved simulations of two-fluid models

- Snapshot of particle volume fraction field – kinetic theory based two-fluid model.
- Squares of different sizes illustrate regions (i.e. filters) of different sizes.

\[
\beta_{fil} = \text{filtered drag coefficient} \\
= \beta_{micro} \left( \bar{\phi}_s |\vec{v}_g - \vec{v}_s| \right)(1 - H)
\]

\[
H = \frac{g \Delta_{fil}}{V_t^2}, \quad \text{parameters characterizing sub-filter scale structure}
\]

\[\Delta_{grid} \ll \Delta_{fil} \ll \Delta_{domain}\]

Igci et al., (2008)
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\[ \beta_{fil} = \text{filtered drag coefficient} \]
\[ = \beta_{micro} \left( \bar{\phi}_s, \left| \vec{v}_g - \vec{v}_s \right| \right) (1 - H) \]
\[ H = f \left( \frac{g \Delta_{fil}}{v_t^2}, \bar{\phi}_s \right) \]

\[ \Delta_{grid} \ll \Delta_{fil} \ll \Delta_{domain} \]

Filtered drag coefficient

\[ \beta_{\text{fil}} = \beta_{\text{micro}} \left( \bar{\phi}_s, \left| \bar{v}_g - \bar{v}_s \right| \right) \left( 1 - H \right); \]

\[ H = f \left( \frac{g \Delta_{\text{fil}}}{V_t^2}, \bar{\phi}_s \right) \]


\( \left\{ \begin{array}{l}
75\mu m \text{ FCC particles} \\
\text{ambient air}
\end{array} \right. \)

\[ \frac{g \Delta_{\text{fil}}}{V_t^2} = 2.056 \Rightarrow \Delta_{\text{fil}} = 1cm \]
Filtered drag coefficient: The present study

\[ \beta_{fil} = \beta_{\text{micro}} \left( \phi_s, \left| \mathbf{\tilde{v}}_g - \mathbf{\tilde{v}}_s \right| \right) (1 - H); \]

\[ H = f \left( \frac{g \Delta_{fil}}{V_t^2}, \phi_s, \ldots \right) \]

\[ \left\{ \begin{array}{l}
75\mu m \text{ FCC particles} \\
\text{ambient air}
\end{array} \right\}
\]

\[ \frac{g \Delta_{fil}}{V_t^2} = 2.056 \Rightarrow \Delta_{fil} = 1cm \]

Filtered drag coefficient: The present study

\[
\beta_{\text{fil}} = \beta_{\text{micro}} \left( \bar{\phi}_s, \left| \bar{\nu}_g - \bar{\nu}_s \right| \right) (1 - H); \quad H = f \left( \frac{g \Delta_{\text{fil}}}{V_t^2}, \bar{\phi}_s, \frac{\bar{\nu}_g - \bar{\nu}_s}{V_t} \right)
\]
Filtered drag coefficient: The present study

\[ \frac{\langle \tilde{v}_{\text{slip}} \rangle}{v_t} = 0.60 \]

- As filter size increases, the filtered drag coefficient decreases.
- Does suggest the existence of large filter size asymptote.

\[ H = f \left( \frac{g \Delta_{fil}}{v_t^2}, \phi_s, \frac{\tilde{v}_g - \tilde{v}_s}{v_t} \right) \]

\[ \begin{array}{c}
\{ 75 \mu m \text{ FCC particles} \\
\{ \text{ambient air} \}
\end{array} \]

\[ \frac{g \Delta_{fil}}{v_t^2} = 2.056 \Rightarrow \Delta_{fil} = 1 cm \]
Filtered drag coefficient: The present study

As slip velocity increases, the filtered drag coefficient decreases.

\[ H = f \left( \frac{g \Delta_{fil}}{v_t^2}, \phi_s, \frac{\tilde{v}_g - \tilde{v}_s}{v_t} \right) \]

\[ \left\{ \begin{array}{l} 75 \mu m \text{ FCC particles} \\ \text{ambient air} \end{array} \right\} \]

\[ \frac{g \Delta_{fil}}{v_t^2} = 2.056 \Rightarrow \Delta_{fil} = 1 cm \]
Filtered drag coefficient: The present study

- As slip velocity increases, the filtered drag coefficient decreases.
- Same trend at different filter sizes.

\[ H = f \left( \frac{g \Delta_{\text{fil}}}{V_t^2}, \phi_s, \frac{\tilde{V}_g - \tilde{V}_s}{V_t} \right) \]

\[ \begin{cases} 75 \mu m \text{ FCC particles} \\ \text{ambient air} \end{cases} \]

\[ \frac{g \Delta_{\text{fil}}}{V_t^2} = 2.056 \Rightarrow \Delta_{\text{fil}} = 1 cm \]
Filtered drag coefficient: The present study

- As slip velocity increases, the filtered drag coefficient decreases.
- NOT THE USUAL INERTIAL CORRECTION!

\[ H = f \left( \frac{g \Delta_{fil}}{V_t^2}, \phi_s, \frac{\tilde{V}_g - \tilde{V}_s}{V_t} \right) \]

\[ \begin{cases} 
75 \mu m \text{ FCC particles} \\
\text{ambient air}
\end{cases} \]

\[ \frac{g \Delta_{fil}}{V_t^2} = 2.056 \Rightarrow \Delta_{fil} = 1cm \]
Filtered drag coefficient: The present study

As the slip velocity increases, the sub-filter scale distribution of particles becomes more segregated

$$H = f \left( \frac{g \Delta_{fil}}{V_t^2}, \phi_s, \frac{\tilde{V}_g - \tilde{V}_s}{V_t} \right) \begin{cases} 75\mu m \text{ FCC particles} \vspace{0.2cm} \cr \text{ambient air} \end{cases} \begin{vmatrix} \frac{g \Delta_{fil}}{V_t^2} \cr = 2.056 \Rightarrow \Delta_{fil} = 1cm \end{vmatrix}$$
Filtered drag coefficient: The present study

At low slip velocities:

\[ H_1 = \begin{cases} 
0, & \bar{\phi}_s < \bar{\phi}_s^c \\
A (\bar{\phi}_s - \bar{\phi}_s^c), & \bar{\phi}_s > \bar{\phi}_s^c 
\end{cases} \]

At high slip velocities:

\[ H_1 = B + A \bar{\phi}_s \]

\[ H = \min(h_{env}, H_1) \]
Filtered drag coefficient: The present study

\[ H = f \left( \frac{g \Delta \text{fil}}{v_t^2}, \phi_s, \frac{\tilde{V}_g - \tilde{V}_s}{v_t} \right) \]
Filtered particle phase viscosity

$$\frac{\mu_s g}{\rho_s V_t^3}$$

Dimensionless effective solid viscosity

- $4.112$
- $8.224$
- $16.448$
- $24.672$

Filtered solid volume fraction

$$\frac{\mu_s g}{\rho_s V_t^3} = f \left( \frac{g \Delta_{fil}}{V_t^2}, \phi_s \right)$$

- $75 \mu m$ FCC particles
- ambient air

$$\frac{g \Delta_{fil}}{V_t^2} = 4.112 \Rightarrow \Delta_{fil} = 2cm$$

Filtered particle phase viscosity: Present study

\[ \overline{S}_i = \sqrt{2 \ddot{S}_i : \ddot{S}_i}, \quad \ddot{S}_i = \frac{1}{2} (\nabla \tilde{\nu}_i + \nabla \tilde{\nu}_i^T) - \frac{1}{3} (\nabla \cdot \tilde{\nu}) I, \quad i = s, g \]

\[ \mu_{fil, i} = \rho_i \Delta_{fil}^2 \ddot{S}_i C_{visc, i}, \quad i = s, g \]

\[ C_{visc, s} = 0.105 \overline{\phi}_s \]

\[ C_{visc, g} = 0.17 - 0.275 \overline{\phi}_s \]
Summary

- A more refined model for the filtered fluid-particle drag force is presented.

- Smagorinsky-like model for the filtered particle and fluid phase viscosities capture the computationally generated data nicely.

- Smagorinsky-like model for the meso-scale particle and fluid phase pressures (akin to turbulent kinetic energy) works nicely as well (not presented).
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Filtered meso-scale pressure: Present study

\[
\overline{S_i} = \sqrt{2 \overline{S_i} : \overline{S_i}}, \quad \overline{S_i} = \frac{1}{2} (\nabla \tilde{\nu}_i + \nabla \tilde{\nu}_i^T) - \frac{1}{3} (\nabla \cdot \tilde{\nu}) I, \quad i = s, g
\]

\[
P_{fil, i} = \rho_i \Delta^2_{fil} \overline{S_i}^2 \left( \frac{g \Delta_{fil}}{V_t^2} \right)^{2/7} C_{press, i}, \quad i = s, g
\]

\[
C_{press, g} = 0.275 - 0.44 \overline{\phi}_s
\]
Filtered meso-scale pressure: Present study

\[ \bar{S}_i = \sqrt{2 \bar{S}_i : \bar{S}_i}, \quad \bar{S}_i = \frac{1}{2} (\nabla \tilde{\nu}_i + \nabla \tilde{\nu}^T_i) - \frac{1}{3} (\nabla \cdot \tilde{\nu}) I, \quad i = s, g \]

\[ P_{fil, i} = \rho_i \Delta^2_{fil} \bar{S}_i^2 \left( \frac{g \Delta_{fil}}{v_t^2} \right)^{2/7} C_{press, i}, \quad i = s, g \]

\[ C_{press, s} = 0.17 \bar{\phi}_s \]