Filtered Two-Fluid Model for Gas-Particle Suspensions

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Festschrift for Professor Dimitri Gidaspow's 75th Birthday – II

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Bayou E (Gaylord Opryland Hotel)
(3:57-4:18 PM)

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Characteristics of flows in turbulent fluidized beds & fast fluidized beds

• Persistent density and velocity fluctuations
  ➢ Wide range of length and time scales

• Identifiable macroscopic inhomogeneous structures
  ➢ Radial variation of particle volume fraction and fluxes

FLOW BEHAVIOR IN FAST FLUID BEDS/RISERS

Why do particles segregate?
Two-fluid model equations

Solids
\[ \frac{\partial (\rho_s \phi_s)}{\partial t} + \nabla \cdot (\rho_s \phi_s u_s) = 0 \]

Fluid
\[ \frac{\partial (\rho_f \phi_f)}{\partial t} + \nabla \cdot (\rho_f \phi_f u_f) = 0 \]

Gidaspow's 1994 book + many papers!

\[ \phi_s + \phi_f = 1 \]

Solids
\[ \frac{\partial}{\partial t} (\rho_s \phi_s u_s) + \nabla \cdot (\rho_s \phi_s u_s u_s) = -\nabla \cdot \sigma_s - \phi_s \nabla \cdot \sigma_f + f + \rho_s \phi_s g \]

Typical closures used widely:
- Newtonian fluid model for gas-phase stress
- Kinetic theory of granular materials for solid phase stress
- Inter-phase force – due to gas-particle drag
75 μm particles in air

Average particle volume fraction: 0.05

2D Domain size: 64 cm x 64 cm

with MFIX (Multiphase Flow with Interphase eXchanges)
Modeling challenge for gas-particle flows

Develop models that allow us to focus on large-scale flow structures, without ignoring the possible consequence of the smaller scale structures.

Original two-fluid model and constitutive relations

**Significant advances in the past three decades**

Filtered two-fluid model

*Modified constitutive relations for hydrodynamic terms*

- species and energy dispersion
- interphase heat and mass transfer rates
- even modified reaction rate expressions!

**Jinghai Li: EMMS approach**
Filter “data” generated through highly resolved simulations of two-fluid models

Snapshot of particle volume fraction fields obtained in highly resolved simulations of gas-particle flows. Squares illustrate regions (i.e. filters) of over which averaging over the cells is performed.

Filtered variables

\[ \phi_s = \bar{\phi}_s + \phi'_s \quad \phi_f = \bar{\phi}_f + \phi'_f \quad \bar{\phi}' = 0 \]

\[ u_s = \bar{u}_s + u'_s \quad \bar{\phi}_s u_s = \phi_s u_s \quad \bar{\phi}_s u'_s = 0 \]

\[ u_f = \bar{u}_f + u'_f \quad \bar{\phi}_f u_f = \phi_f u_f \quad \bar{\phi}_f u'_f = 0 \]

All filtered quantities are functions of space and time.
Filtered particle phase momentum balance

\[
\frac{\partial}{\partial t} \left( \rho_s \phi_s u_s \right) + \nabla \cdot \left( \rho_s \phi_s u_s u_s \right) = -\nabla \cdot \sigma_s - \phi_s \nabla p_f + f_{\text{drag}} + \rho_s \phi_s g
\]

Upon filtering,

\[
\frac{\partial}{\partial t} \left( \rho_s \phi_u \overline{u} \right) + \nabla \cdot \left( \rho_s \phi_u \overline{u} \overline{u} \right) = -\nabla \cdot \left( \overline{\sigma_s} + \rho_s \phi_u \overline{u}' \overline{u}' \right) - \phi_s \nabla p_f
\]

\[
- \phi_s' \nabla p_f' + f_{\text{drag}} \quad + \quad \rho_s \phi_s g
\]

stress due to sub-filter scale fluctuations

effective sub-filter scale fluid-particle interaction force
Sub-filter scale correlations

\[ \rho_s \phi_s u'_s u'_s + \sigma_s = p_{s,\text{eff}} I + 2 \mu_{s,\text{eff}} S \]

Scaled filter size

\[ \frac{g \Delta}{V_t^2} = \frac{1}{Fr_{\Delta}} \]

All the effective quantities will depend on the choice of filter size, \( \phi_s \) and so on; such filter size dependence is present in LES of turbulent flows as well.
Filtered drag coefficient decreases as filter size increases for both 2-D and 3-D

\[ \frac{\beta_{\text{eff}} V_t}{\rho_s g} \]

Example: 75 \( \mu \text{m} \); 1500 kg/m\(^3\); domain size = 8 cm

\[
\frac{g \Delta}{V_t^2} = \frac{1}{Fr_\Delta}
\]

\[
\frac{1}{Fr_\Delta} = 2 \implies \Delta = 1 \text{ cm}
\]
Wall correction to the filtered closures

- 2-D Kinetic theory based simulations
- Height: 500 cm
- Width: 20 cm, 30 cm, 50 cm
- The inlet gas superficial velocity: 93 cm/s.
- The inlet particle phase superficial velocity is 2.38 cm/s.
- The inlet particle phase volume fraction is 0.07.
- Partial-slip BC for particle phase and free-slip BC for gas phase

Grid size: 0.25 cm (0.514 dimensionless units)
The filtered drag coefficient is noticeably different in the core and the wall regions. The wall correction for the filtered drag coefficient is independent of channel width and filter size (not shown). The same conclusion applies for filtered particle phase pressure and viscosity.
Verification of the filtered model

Original two-fluid model  \rightarrow  Filtered two-fluid model

Solve a test problem using the original two-fluid model equations

Solve the same test problem using the filtered two-fluid model equations

Compare
Verification of the filtered model

- Compare predictions of kinetic theory and filtered models
  - Height: 500 cm
  - Width: 30 cm
  - The inlet gas superficial velocity: 93 cm/s.
  - The inlet particle phase superficial velocity is 2.38 cm/s.
  - The inlet particle phase volume fraction is 0.07.
- Free-slip boundary conditions

75 μm particles in air

Splash plate

15 cm

500 cm

Width
Mass inventory of particles in the riser

The minimum grid size required for:

- Kinetic theory model:
  Grid size: 0.25-0.50 cm

- 2cm-filtered model:
  Grid size: 1.00 cm

Grid size = \( \frac{1}{2} \) Filter size or smaller

Increasing resolution
Solution of discretized form of the filtered two-fluid model

Filtered model with closures corresponding to a filter size of 2 cm

Nearly grid-size independent time-averaged profiles when grid size is less than or equal to one-half of the filter size.
Solution of discretized form of the filtered two-fluid models

Compare predictions of filtered models with different filter sizes

Filtered model with closures corresponding to filter sizes of 2 and 4 cm yield nearly the same time-averaged results as long as grid size is less than equal to 0.5 (filter size)
Summary

• Performed highly resolved simulations of a kinetic theory based two-fluid model for gas-particle flow in various test domains

• Filtered the results to learn about constitutive relations for the filtered two-fluid model

• Verified the fidelity of the filtered model

• Validation remains to be completed – in progress.
• Filtered species and energy balance equations (and rate of chemical reactions) remain to be developed: future research project.

Dear Dimitri, Happy 75th!